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Rational Expectations, Supply Effect, and Stock Price Adjustment Process: A Simultaneous Equation Approach

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Abstract

In this paper, following Black (1976) a dynamic capital asset pricing model in terms of rational expectations is theoretically derived. This new theoretical model is compared with previous capital asset pricing models. It is shown that the theoretical asset pricing model derived in this paper is a generalized case of Cheng and Grauer's (1980) model and Cheng and Grauer's model might be misspecified.

A. Introduction

In a general equilibrium model, prices of both riskless and risky securities are generally determined by the interaction of the supply and demand. Both expected and unexpected information would affect both the supply and demand of the security market and thereby influence the probability distribution of the returns on securities. In addition, investors' expectations on security demand and supply might be different; therefore, the supply and demand of securities is a pivotal determinant for capital asset pricing.

The capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) [SLM] is a single-period equilibrium analysis in which the behavior of security demand, conditional upon a postulated probability distribution of returns, is examined. Three major assumptions are that: (i) an investment is considered as a one-period activity to be liquidated at the end of the period, (ii) returns are serially independent and obey a stationary distribution, and (iii) the supply of various risky securities are fixed. SLM type of CAPM is a single period instead of a multi-period model. In a single period context the last two assumptions above are less critical. At the end of the period, ex post returns are drawn from the ex ante joint return distribution. Investors would not be troubled by rebalancing their portfolios depending on their beliefs and opportunities for the next period. In the multi-period scenario, however, investors take the proceeds of their investments at the end of the period and change their shares of wealth invested in various securities to maximize expected utilities. Thus attempting to apply the single period CAPM in a multi-period setting implies that the investors would have to behave in an irrational fashion basing their portfolio decisions on ex ante beliefs that are never realized. However, supply is generally not fixed since firms would like to issue or retire their securities to take advantage of changes in relative prices of securities to maximize the market values of their firms. In other words, a sudden stochastic change in a firm's opportunity set is a perturbation on the multi-period equilibrium, and therefore, the distribution of returns must change.

The purpose of the paper is to derive the stochastic nature of security prices in the multi-period context, allowing returns to be endogenous to both demand and supply decisions by firms as assumed by Black (1976). The model will show that expectation error of a security price due to unexpected random shock over certain period is related to the adjustments of prices and dividends of all other securities. It thus resembles the model by Cheng and Grauer (1980) in the sense that the market portfolio no longer appears explicitly in the model and instead, one security price is associated with other security prices directly. Overall, Cheng and Grauer's empirical results clearly shows that two arbitrarily chosen security prices cannot fully explain a security price. In addition, Cheng and Grauer found that there was a statistically significant increase in the adjusted R-square (coefficient of determination) as the number of regressions (i.e., prices) increased. This paper will theoretically and empirically generalize Cheng and Grauer's model in which one security price can be perfectly explained by any two other security prices.

Section B develops a multi-period equilibrium capital asset pricing model in terms of rational expectations. Both supply of and demand for security equations are derived. Section C derives a structural-form model for re-examining Cheng-Grauer's (1980) propositions in testing capital asset pricing. Section D derives a structural form model for testing the existence of supply effect for capital asset pricing. Section E summarizes the results of the paper.

B. A Multi-Period Equilibrium Capital Asset Pricing Model

B.1 Demand for Securities

In deriving capital asset pricing models, it is assumed that individual investors hold both risk-free and risky securities. If individual investors make their plans at time t for the time t+1, subject to the constraints of their initial wealth, given information that is available at time t. Then, they have a single-period planning horizon to maximize the expected utility of terminal wealth. At time t, firms are assumed to announce both investment and financing policies for the end-of-period cash flow and, at the same time, investors are assumed to reconstruct their portfolios in accordance with the new price and quantity vector of all the securities in the market.

The SLM type of CAPM describes an equilibrium relationship in which, for a given period in a mean-variance optimizing world, expected return of a security is linearly related to its systematic risk. However, there is no guarantee from the theory that the risk-return relationship is stationary over time. In this kind of model, the expected return and variance-covariance structure of these returns are exogenous while their systematic risks and the return on the market portfolio, or the return on the zero-beta, are endogenous. Empirically, SML type of CAPM assumes that (i) the market model holds in every period, (ii) the CAPM holds in every period, and (iii) the beta is stable through time.

Cheng and Grauer (1980) argue that an assumption of stationarity of beta in the SLM type CAPM implies that all prices would have to move in equal proportions over time. To circumvent this problem, they express the model as

$$p_t = \gamma_t^{-1}(\Sigma_t^{-1}\bar{r}_t - \bar{r}_{zt}\Sigma_t^{-1}q), \tag{1}$$

where p_t is a column vector of asset prices, γ_t is the market price of risk, Σ_t is the covariance matrix of asset returns, $\bar{r}_t$ is a column vector of expected asset returns, $\bar{r}_{zt}$ is the expected returns of the zero-beta portfolio, and q is a column vector of ones. Cheng and Grauer argue that the assumption of stationary-return distribution is less restrictive than the stationarity of the beta. Equation (1) can be rewritten in the form:

$$p_t = \gamma_t^{-1}(\Sigma^{-1}\bar{r} - \bar{r}_{zt}\Sigma^{-1}q), \tag{2}$$

In equation (2), the return of zero-beta portfolio is still time dependent. By eliminating γ_t^{-1} and $\bar{r}_{zt}$, one at a time, they derive the final testable form in terms of price per share as

$$p_{it} = b_j p_{jt} + c_k p_{kt}, \tag{3}$$

where b_j and c_k are time-independent coefficients. Therefore, a test of the CAPM with the assumption of stationarity requires only three security prices.

Turnbull and Winter (1982) and Sweeney (1982) argue that stationarity of return distribution is inconsistent with nonstationarity of the zero-beta return: that is, if the zero-beta return is nonstationary the expected returns and covariances must also be time dependent.

The expected returns and covariances depend on relative prices which will generally change over time. Any change in the observed current equilibrium prices will induce a change in the return-generating process, conditional upon this new set of prices. The point is that an assumption of stationarity in returns is an assumption on an endogenous variable in a general equilibrium context. In the present model, the covariance matrix is assumed to be constant over time, but the vector of the expected returns is determined within the model. As a logical extension, supply side will be introduced in the price determination process.

Under the standard assumptions of the SLM CAPM including unlimited short sales or borrowing of risk-free security, the investor's objective is to maximize the expected utility function. A negative exponential function for the investor's utility of wealth is assumed:

$$U = a - he^{-bW_{t+1}}, \tag{4}$$

where W_{t+1} = end-of-page wealth, e = base of natural logarithm, a, b, and h = constants.

The dollar returns on n marketable risky securities are:

$$x_{j,t+1} = p_{j,t+1} - p_{jt} + d_{j,t+1}, \quad j = 1,\ldots,n, \tag{5}$$

where $p_{j,t+1}$ = (random) price of security j at time t+1, p_{jt} = price of security j at time t, and $d_{j,t+1}$ = (random) dividend or coupon on security j at time t+1, are jointly normally distributed.[1] Although equation (5) is similar to the Black's (1976) framework, the dividend or coupon is allowed, in this model, to change over time while Black holds it constant. Expected returns are, by the "rational" expectations,

$$E_t x_{j,t+1} = E_t p_{j,t+1} - p_{jt} + E_t d_{j,t+1}, \quad j = 1,\ldots,n, \tag{6}$$

where $E_t p_{j,t+1} = E(p_{j,t+1}|\Omega_t)$, $E_t d_{j,t+1} = E(d_{j,t+1}|\Omega_t)$; Ω_t is given information available at time t.

Then a typical investor's expected value of end-of-period wealth is

$$\overline{W}_{j,t+1} = W_t + r^*(W_t - q'_{t+1}p_t) + q'_{t+1}\overline{x}_{t+1}, \tag{7}$$

where $p_t = (p_{1t},p_{2t},\ldots,p_{nt})'$, $x_{t+1} = (x_{1,t+1},x_{2,t+1},\ldots,x_{n,t+1})' = E_t p_{t+1} - p_t + E_t d_{t+1}$, $q_{t+1} = (q_{1,t+1},q_{2,t+1},\ldots,q_{n,t+1})'$, $q_{j,t+1}$ = number of units of security j after the reconstruction of his portfolio, r^* = (nonstochastic, scalar) risk-free rate, prime denotes transpose.

The second term in equation (7) is the return on the risk-free investment and the last one is the return on the portfolio of risky securities. The variance of W_{t+1} is:[2]

$$V(W_{t+1}) = E(W_{t+1} - \overline{W}_{t+1})(W_{t+1} - \overline{W}_{t+1})' = q'_{t+1}Sq_{t+1}, \qquad (8)$$

where $S = E(x_{t+1} - \overline{x}_{t+1})(x_{t+1} - \overline{x}_{t+1})'$ = covariance matrix of returns of risky securities that is symmetric positive definite, assumed to be constant.

Maximization of the expected utility of W_{t+1} is equivalent to:

$$\max.\ \overline{W}_{t+1} - (b/2)V(W_{t+1}), \qquad (9)$$

or substituting equation (7) and equation (8),

$$\max_{q_t}.(1+r^*)W_t + q'_{t+1}(\overline{x}_{t+1} - r^*p_t) - (1/2)q'_{t+1}Sq_{t+1}. \qquad (10)$$

The solution becomes:

$$q_{t+1} = b^{-1}S^{-1}(\overline{x}_{t+1} - r^*p_t). \qquad (11)$$

Assuming homogeneous expectations regarding $\overline{x}_{t+1}$ and S the aggregate demand for risky securities over m investors is:

$$Q_{t+1} = \sum_{k=1}^{m} q^k_{t+1} = cS^{-1}(E_tp_{t+1} - (1+r^*)p_t + E_td_{t+1}), \qquad (12)$$

where $c = \Sigma(b^k)^{-1}$.[3]

B.2 <u>Supply of Securities</u>

In this section, an endogenous supply side to the model is now derived. There are N firms in the market. At the end of period t, the firm i holds an accumulated stock of physical capital k_{it} with unit price p^k_{it}. The firm plans ahead one period and determines, by means of an optimization procedure to be discussed below, a desired stock of capital $k_{i,t+1}$, which it wishes to hold for the period t+1.

Assuming that the desired capital is equal to the actual capital, in the absence of depreciation, its investment plan is specified by

$$I_{i,t+1} = p^k_{it}(k_{i,t+1} - k_{it}), \qquad i = 1,\ldots,N \tag{13}$$

which is financed by issuing securities. N is the total number of firms and is equal to or smaller than the number of securities n. Thus, if the market for physical capital is in equilibrium the supply of risky securities is determined by the demand for the physical capital. Thus,

$$p_{it}\Delta_{i,t+1} = p^k_{it}\Delta k_{i,t+1}, \tag{14}$$

where Q_{it} is a n_i dimensional quantity vector of firm i's securities.

Sandmo (1971), Leland (1972), and Holthausen (1976) have extended the standard price theory of the competitive firm under certainty to the world of uncertainty. The literature makes the assumption (either implicitly or explicitly) that the firm is required to make all its production decisions for a given period before the selling price is known. The price of output is a nonnegative random variable whose distribution is subjectively determined by the firm's expectations. The authors assume that the firm is maximizing the expected utility of profits. The result shows that the optimal output for the competitive firm under price uncertainty is characterized by marginal cost being less than the expected price. Thus, under price uncertainty, output will be smaller than the certainty output and consequently, at the optimum level of output, the expected price is greater than average cost. This implies that the firm requires strictly positive expected profit in order to operate in a competitive environment under price uncertainty.

There have been many theoretical and empirical studies of capital structure and hence of the cost of capital. Modigliani and Miller (1958) argued that, in a perfect market, the value of a firm is unaffected by its capital structure. Later, in the modified model with corporate taxes, they argued that the firm could increase its value by borrowing due to the tax-deductibility of interest expense to the bondholders. However, this is certainly another extreme since we cannot find any one firm with extreme debt ratio in the real world.

The tax rate on long-term capital gains is lower than that on dividend income. Thus, the personal tax is higher on interest than on equity income. Due to this tax differential, corporate borrowing may reduce firm value. In other words, up to some point, at which one minus corporate tax rate is equal to one minus personal tax on equity income multiplied by one minus personal tax on interests, corporate borrowing is cheaper in terms of after-tax total income to all investors including bondholders and stockholders as well. The reason is that when firms start borrowing investors in low income tax bracket would be willing to hold debt. As they borrow more, firms should be able to offer investors in higher tax brackets higher interest rates which can compensate for the personal tax loss through corporate tax loss through corporate tax saving in order to persuade them to buy bonds. Borrowing would stop when the corporate tax saving equals the personal tax loss. Therefore, there is an optimal leverage ratio for firms as a whole, but not for a single firm because once the low-bracket taxpayers have invested in bonds no single firm can benefit by borrowing more. This is what Miller (1977) has shown.

There are still some other factors to be considered. If direct or indirect bankruptcy costs are material in the magnitude they should reduce the value of the firm as the firm borrows more. If the firm's management took actions that would benefit stockholders at the expense of bondholders it would certainly undermine the bondholders' wealth position. This is why bonds are protected by restrictive covenants on investment decisions or financing and dividend policy. These agency costs should also reduce the value of the firm.

It is assumed that there exists a solution to the optimal capital structure and that the firm has determined the optimal level of additional investment. The one-period objective of the firm is to achieve the minimum cost-of-capital vector with adjustment costs involved in changing the quantity vector $Q_{i,t+1}$ to respond to change in demand for risky securities by the investors:[4]

$$\text{min. } E_t d'_{i,t+1} Q_{i,t+1} + (1/2)\Delta Q'_{i,t+1} Q_i \Delta Q_{i,t+1},$$

$$\text{subject to } p_{it} \Delta Q_{i,t+1} = p^k_{it} \Delta k_{i,t+1}, \tag{15}$$

where A_i is an $n_i \times n_i$, $\sum_{i=1}^{N} n_i = n$, positive definite matrix of coefficients measuring the assumed quadratic costs of adjustment. The solution to equation (15) is:

$$\Delta Q_{i,t+1} = A_i^{-1}(\lambda_i p_{it} - E_t d_{i,t+1}), \tag{16}$$

where λ_i is the scalar lagrangian multiplier.

Aggregating equation (16) over N firms, the final form of the supply function is given by

$$\Delta Q_{t+1} = A^{-1}(Bp_t - E_t d_{t+1}), \tag{17}$$

where

$$A^{-1} = \begin{bmatrix} A_1^{-1} & & & \\ & A_2^{-1} & & \\ & & \cdot & \\ & & & A_N^{-1} \end{bmatrix}, \quad B = \begin{bmatrix} \lambda_1 I & & & \\ & \lambda_2 I & & \\ & & \cdot & \\ & & & \lambda_N I \end{bmatrix}, \text{ and } Q = \begin{bmatrix} Q_1 \\ Q_2 \\ \cdot \\ Q_N \end{bmatrix}.$$

To get the nonstochastic matrix B in equation (17), it is also assumed that the λ_i are constant over time. Equation (17) implies that the amount of each security newly issued is positively related to its own price and is negatively related to its required return.

B.3 Multi-Period Equilibrium

The model is summarized by the following equations:

$$Q_{t+1} = cS^{-1}(E_t p_{t+1} - (1+r^*)p_t + E_t d_{t+1}), \tag{18}$$

$$\Delta Q_{t+1} = A^{-1}(Bp_t - E_t d_{t+1}). \tag{19}$$

Combining equation (18) and equation (19) shows that

$$cS^{-1}(E_t p_{t+1} - E_{t-1}p_t - (1+r^*)p_t + (1+r^*)p_{t-1}$$
$$+ E_t d_{t+1} - E_{t-1}d_t) = A^{-1}(Bp_t - E_t d_{t+1}) + u_t, \tag{20}$$

which is a second-order system of stochastic difference equations in the random vector p_t, and conditional expectations $E_{t-1}p_t$ and $E_{t-1}d_t$. The random disturbance to this equation u_t is added to take into account possible discrepancies in the system and assumed expected value of zero and non-autocorrelations. Rearranging (20), a standard structural form of a simultaneous equation system is obtained:

$$G(L)p_t + \sum_{i=1}^{2} W_i E_{t-i} p_{t+1-i} + \sum_{i=0}^{1} \Gamma_i E_{t-i} d_{t+1-i} = u_t, \qquad (21)$$

where $G(L) = G_0 + G_1 L$, L is lag variable,

$G_0 = -(1+r^*)cS^{-1} - A^{-1}B,$

$G_1 = (1+r^*)cS^{-1},$

$W_0 = cS^{-1},$

$W_1 = -cS^{-1},$

$\Gamma_0 = cS^{-1} + A^{-1},$

$\Gamma_1 = -cS^{-1}.$

All coefficients are n x n matrices.

Equation (21) includes one lagged price, one-period and two-period expectations of prices, one-period and two-period expectations of dividends. According to Wallis (1980), Revanker (1980), Hoffman and Schmidt (1981), and Pesaran (1981), the general solution for equation (21) cannot be found. Therefore, to set up a testable form of equation (21), the conditional expectations upon information Ω_{t-1} available as of time t-1 are taken:

$$G_1 p_{t-1} + (G_0 + W_1)E_{t-1}p_t + W_0 E_{t-1} p_{t+1}$$

$$- \Gamma_0 E_{t-1} d_{t+1} + \Gamma_1 E_{t-1} d_t = u_t, \qquad (22)$$

which is based on the following well-known properties:

$$E_{t-1}E_t p_{t+1} = E[E(p_{t+1}|\Omega_t)|\Omega_{t-1}]$$

$$= E(p_{t+1}|\Omega_{t-1})$$

$$= E_{t-1}p_{t+1}, \text{ where } \Omega_{t-1} \subset \Omega_t.$$

Subtracting equation (22) from equation (21) ,

$$G_0(p_t - E_{t-1}p_t) + W_0(E_t p_{t+1} - E_{t-1}p_{t+1})$$

$$+ \Gamma_0(E_t d_{t+1} - E_{t-1}d_{t+1}) = u_t. \tag{23}$$

Equation (23) shows that prediction errors in prices due to unexpected disturbances are a function of expectation adjustments in prices and dividends two-periods ahead. This is closely related to the holding-period-return concept. The holding period return at time t-1 depends mainly on the expected price and dividend one-period ahead. However, the actual price at time t would be determined again by the expected price and dividend at time t+1. This process may continue infinitely. However, the model indicates that investors look forward to prices and dividends two-periods ahead. If the actual price at time t deviates from the price expected from time t-1, the difference, that is forecast error, is equal to the sum of changes in expectations about prices and dividends at time t+1 due to the change in information available between time t-1 and t. Moreover, the differences are all simultaneously determined. In other words, forecast error of security j is determined not only by expectation adjustments in its own price and dividend, but by those in prices and dividends of other securities. Therefore, equation (23) is a generalized capital asset pricing model which will be used to re-examine Cheng and Grauer's test of the CAPM in Section C.

Another implication of the model (23) is that the supply-side effect can be seen by substituting the definitions of coefficient matrices in the equation:

$$-cS^{-1}1+r^*) - A^{-1}B(p_t - E_{t-1}p_t) + cS^{-1}(E_tp_{t+1} - E_{t-1}p_{t+1})$$

$$+ (cS^{-1} + A^{-1})(E_td_{t+1} - E_{t-1}d_{t+1}) = u_t, \qquad (24)$$

Were the adjustment costs very large, they would keep the firms from seeking to raise new funds or to retire old securities. This would cause the matrix A to vanish in equation (24), reducing the model to a certainty equivalent relationship:

$$p_t - E_{t-1}p_t = (1+r^*)^{-1}(E_tp_{t+1} - E_{t-1}p_{t+1})$$

$$+ (1+r^*)^{-1}(E_td_{t+1} - E_{t-1}d_{t+1}) + v_t, \qquad (25)$$

where $v = -c^{-1}Su_t$. Equation (25) suggests that current forecast error in a price is determined only by the sum of the discounted values, at one plus risk-free rate, of the expectation adjustments in its own next-period price and dividend. The coefficients will be all equal across securities although the disturbances are contemporaneously correlated. This implication will be tested in Section D using various regression methods.

C. Structural-Form Approach to Test the CAPM

Implications of equation (23) will be empirically estimated and analyzed in Sections C and D. The Box-Jenkins transfer-function modelling, a method to obtain the conditional expectations is explained in Section C.1. In Section C.2, a new structural approach is proposed to test the capital asset pricing model.

In estimating the model, it is assumed that (1) there are ten groups of firms in the market which can be represented by the ten portfolios grouped according to payout ratios, (2) each firm issues only one kind of security, common stock, (3) economically rational expectations are based on the information subset which only consists of price, dividend, and earnings, (4) dividend is influenced by earnings, and (5) dividend is exogenous in the model, implying that dividend is not influenced by price. The first assumption is needed to reduce the number of securities to a controllable size and the second to make the matrices A and B of equation (17) in the model diagonals. By the third assumption, emphasis can be given to only those three variables and by the fourth, past values of earnings can be used to forecast dividends, in addition to past observations of dividends.

C.1 Time Series Forecasting

To estimate model (23), proxies must be obtained for one-period-ahead and two-period-ahead conditional expectations for dividend and price. Based on the empirical findings of the Granger (1977) causality tests, dividend or earnings may not be used to reduce the forecast errors of the future values of price. Thus, the conditional expectations of price are taken based on its own past history only. From the univariate time series analysis, integrated moving average process, or a random walk model in a more familiar term, fits price series:

$$(1 - B)p_t = a_t,$$

where a_t is a white noise. It follows that $\hat{p}_{t-1}(1) = p_{t-1}$, $\hat{p}_{t-1}(2) = p_{t-1}$, $\hat{p}_t(1) = p_t$, where $\hat{p}_t(1)$ is one-period-ahead forecast $\hat{p}_t(k)$, is the condition expectation of p_{t+k}, given the history of the series up to time t:

$$p_t(k) = E(p_{t+k}|H_t) = E(p_{t+k}|\ldots,p_{t-1},p_t)$$

where H_t is the past history of p_t and E denotes expectation. Thus,

$$p_t - \hat{p}_{t-1}(1) = \hat{p}_t(1) - \hat{p}_{t-1}(2) = p_t - p_{t-1}. \tag{26}$$

Equation (23) becomes, using the above result,

$$(G_0 + W_0)(p_t - p_{t-1}) + \Gamma_0(E_t d_{t+1} - E_{t-1}d_{t+1}) = u_t. \tag{27}$$

To improve one- and two-period-ahead forecasts of dividends, past realizations of earnings as well as those of dividends are used. If the same set of exogenous forces which are particularly responsible for trend, seasonality, and sudden interventions underlies the two time series, then a bivariate model of the relationship may incorporate these forces indirectly. In fact, at this stage, another "filter" (earnings) in addition to the random shock is utilized, combined together to generate the output series (dividend).

In this section, unidirectional causality model which is called the transfer function model extensively discussed by Box and Jenkins

(1976) is identified and estimated. However, this model becomes inappropriate when there is a feedback between the two series as Granger and Newbold (1977) point out.[5]

Transfer function model is a time series technique to use another time series x as a linear filter to produce the output series y:

$$
\begin{aligned}
y_t &= v_0 x_{t-b} + v_1 x_{t-b-1} + v_2 x_{t-b-2} + \cdots \\
&= (v_0 + v_1 B + v_2 B + \ldots) x_{t-b} \\
&= v(B) x_{t-b}, \qquad (28)
\end{aligned}
$$

where the operator v(B) is called the transfer function of the filter. The equation (28) may be written in an ARIMA form:

$$(1 - \delta_1 B - \ldots - \delta_r B^r) y = (\omega_0 - \omega_1 B - \ldots - \omega_s B^s) x_{t-b},$$

or

$$\delta(B) y_t = \omega(B) x_{t-b}. \qquad (29)$$

Comparing (29) with (28), the transfer function for this model is

$$v(B) = \delta^{-1}(B)\omega(B). \qquad (30)$$

In practice, the system between the input and the output will be infected by disturbance or noise. Thus, the combined transfer function-noise model can be written as:

$$y_t = \delta^{-1}(B)\omega(B) x_{t-b} + \eta_t, \qquad (31)$$

where η_t is noise which is assumed to be generated by an ARIMA process that is statistically independent of the input x_t. The same iterative identification, estimation, and diagnostic checking strategy used for univariate time series modelling is applied in a transfer function model building.

As the autocorrelation function is used to identify within-series correlation in the univariate model, so is the cross-correlation function used to identify between-series correlation. The cross-correlation function is determined by the v-weights in equation (28), the variances of the two time series, and, if the input series is not a white noise, the autocorrelation function of the input time series. This contamination by autocorrelations can be theoretically removed by solving the m-equation system, where m is the length of lag for which the cross-correlation function is computed. However, considerable simplification in the identification process would occur if the input to the system were a white noise. This simplification is possible by prewhitening.

As a first step, a univariate ARIMA model is constructed for the input series:

$$\phi_x(B)\theta_x^{-1}(B)x_t = \alpha_t, \tag{32}$$

which transforms the input series x_t to a serially uncorrelated white noise series α_t. At the same time, an estimate of variance of α_t from the sum of squares of the α's can be obtained. Then, the univariate ARIMA model for the input is inverted and applied to the output y_t:

$$\beta_t = \phi_x(B)\theta_x^{-1}(B)y_t. \tag{33}$$

Notice that β_t may not necessarily be a white noise. Now the model (28) becomes:

$$\beta_t = v(B)\alpha_{t-b} + \varepsilon_t, \tag{34}$$

where ε_t is defined by

$$\varepsilon_t = \phi_x(B)\theta_x^{-1}(B)\eta_t. \tag{35}$$

Multiplying (34) on both sides by α_{t+k} and taking expectations,

$$\gamma_{\alpha\beta}(k) = v_k\sigma_\alpha^2, \tag{36}$$

where $\gamma_{\alpha\beta}(k) = E(\alpha_{t-k}\beta_t)$ is the cross-covariance between α and β at lag +k. Thus,

$$\begin{aligned} v_k &= \gamma_{\alpha\beta}(k)/\sigma_\alpha^2 \\ &= \rho_{\alpha\beta}(k)\sigma_\beta/\sigma_\alpha. \end{aligned} \tag{37}$$

In equation (37), there is no contamination by input autocorrelations and thus, the cross-correlation is directly proportional to the v-weight which defines the bivariate relationship between the two time series. In practice, of course, sample estimates for cross-correlations are used instead of theoretical counterparts.

Next step is to estimate parameters for the transfer function component identified above and, from its residuals, an ARIMA model is identified for the noise component. The final model takes a form

$$y_t = \delta^{-1}(B)\omega(B)x_{t-b} + \phi^{-1}(B)\theta(B)a_t, \tag{38}$$

where a_t is a white noise. Then, the parameters of the whole model are estimated. If the parameters of either component are not statistically significant and otherwise acceptable, a new model must be identified. The final step is diagnosis. If the autocorrelation of residuals of the tentative model has significant spikes at the first and second lags, then noise component must be reidentified. If residuals of the model is correlated with the prewhitened input variable, a new transfer function component must be identified.

Transfer function models were identified and estimated for four individual portfolios. For the other portfolios, there was no significant spike in the cross-correlations which exceeds ±2 standard errors. The estimation results are given in Table 1 along with the univariate estimations. They all satisfied both diagnostic criteria. In all cases, residual mean squares are smaller for transfer function models than for univariate models. This is what was expected. To see if transfer function models also perform better in prediction mean squares of forecast errors were computed. For portfolio 5, average squared forecast errors are substantially larger in transfer function models than in univariate models. There is no net gain or loss for portfolios 6 and 9. Only for portfolio 1, transfer function model offers substantial reduction in forecast errors measured by mean square errors. This is true for both one- and two-period-ahead forecasts.

Based on the results of transfer function model building, various univariate models were employed to obtain the conditional expectations of dividends for all the portfolios except for the portfolio P01 in which a transfer function model was estimated. The combined one- and

TABLE 1

Comparison Between Univariate Model and Transfer Function Model

		MSE		
			Forecast	
Portfolio	Model	Estimation	(+1)	(+2)
1	U: $(1-\underset{(6.84)}{.7013}B)(1-B)d_t = a_t$	.000463	.001551	.004008
	T: $(1-B)d_t=(\underset{(3.12)}{.1428}B^3+\underset{(3.31)}{.1550}B^4+\underset{(2.91)}{.1325}B^6)(1-B)e_t+a_t'$	.000353	.001075	.001244
5	U: $(1-\underset{(9.29)}{.8165}\ )(1-B)d_t=a_t$	.000103	.000128	.000681
	T: $(1-B)d_t=(\underset{(2.53)}{.1258}+\underset{(4.11)}{.2116}B^3)(1-B)e_t+\frac{a_t'}{(1-\underset{(6.12)}{.6778}B)}$	.000067	.000327	.001216
6	U: $(1-\underset{(26.74)}{.9751}B)(1-B)d_t=(1-\underset{(4.58)}{.5770}B^4)a_t$	.000069	.005941	.025817
	T: $(1-B)d_t=(\underset{(4.40)}{.2589}B+\underset{(3.95)}{.2331}B^5)(1-B)e_t+\frac{(1-\underset{(2.49)}{.4012}B^4)a_t'}{(1-\underset{(8.99)}{.8435}B)}$	.00046	.006173	.024782
9	U: $(1-\underset{(3.91}{.4515}B^2)(1-B)d_t = (1-\underset{(4.15)}{.4869}B^8)a_t$	.002172	.021869	.022903
	T: $(1-B)d_t = (\underset{(3.92)}{.2578}+\underset{(3.99)}{.1804}B^3+\underset{2.15}{.1502}B^4)(1-B)e_t + (1-\underset{(15.74)}{.7877}B^8)a_t'$	.001459	.022136	.022005

U = Univariate model.
T = Transfer function model.
MSE = Mean Square Error.
Numbers in parentheses are t-values.

two-period-ahead forecasts are the data to be analyzed in the next section.

C.2 Structural-Form Approach to Test the Capital Asset Pricing Model

As discussed in Section B, Cheng and Grauer (1980) argued that the SLM model, under the assumption of stationary return distribution, implies a linear structure of equilibrium security prices. Their model can be written as:

$$p_{it} = b_j p_{jt} + c_k p_{kt}, \tag{39}$$

where p_{it}, p_{jt}, and p_{kt} are any distinct security prices and b_j, c_k are constants. Equation (39) shows that a security price is linearly related to any two other arbitrarily chosen security prices. To test their model, they set up a generalized regression equation of the form:

$$p_{it} = a_0 + a_1 p_{1t} + \dots + a_k p_{kt} + e_{it}, \tag{40}$$

where e_{it} is the random error term.

They first postulated the following three hypotheses, with $k = 2$ in equation (40): (1) the intercept equals zero, (2) the slope coefficients differ significantly from zero, (3) the adjusted R-square should be near one. With $k > 2$, two more hypotheses were added: (4) the intercept should remain insignificantly different from zero as more regressors are added into equation (40) and (5) the adjusted R-square value should remain near one as k increases from 2 in equation (40). These hypotheses should hold jointly.

They grouped the total sample into twenty portfolios based on individual security betas. Then, in order to test first three hypotheses, portfolios 1, 2, 20 were selected, one at a time, as the dependent variable. Regressions containing two independent variables were run for nine different sets of the independent variables.[6] The results could hardly be said to support the joint hypotheses 1, 2, and 3.

In testing the last two hypotheses, they first ran regressions of equation (40), increasing the number of independent variables from two to nineteen. Then, the estimated intercepts and the adjusted R-squares were regressed on the time indicator, respectively. This was done for each of the three dependent-variable portfolios. The results showed that there were statistically significant trends in the estimated values of the intercept and the adjusted R-square. That is, there was a significant increase in the adjusted R-square as more regressors were added. Cheng and Grauer concluded that this provided evidence against the SLM CAPM.

However, from their model specification, these results should be expected with regard to the model (23), which clearly shows that a security price (or more exactly forecast errors in price) is related to all other security prices and dividends (or more exactly expectation adjustments in dividends). Contrary to their hypotheses 3 and 5, the adjusted R-square values in the regressions with only two independent variables cannot be, on average, near one and consequently, should increase as more regressors are added.

The regressions results based on equation (40) with two regressors are presented in Table 2. Prices are first-differenced following equation (27): $p^*_{it} = p_{it} - p_{i,t-1}$, $p^*_{jt} = p_{jt} - p_{j,t-1}$, and $p^*_{kt} = p_{kt} - p_{k,t-1}$. For each of ten portfolios as the dependent variable, independent variables are portfolios j and k, where $j = 1, 2, 3, 4, 5$ and $k = 11-j$, $j \neq i$, $k \neq i$.[7] For example, portfolio 1 was regressed on portfolios 2 and 9, 3 and 8, 4 and 7, and 5 and 6. Table 2 gives the summary statistics in four regressions for each portfolio, forty in total.

Consider Hypothesis 1: the intercept should not differ from zero. A glance at Table 2 shows that absolute values of the t-statistics are unambiguously smaller than 2. This is drastically different from Cheng and Grauer's finding that absolute values of t-statistics exceeded 2 in 82 percent of entire regressions. Turn to Hypothesis 2: the slope coefficient should be different from zero. 30 percent (12 out of 40) of the $\hat{a}_1$ and as much as 60 percent (24 out of 40) of the $\hat{a}_2$ are found not significantly different from zero. In only 25 percent of the entire regressions, both coefficients are significantly different from zero. Cheng and Grauer found 21 percent of the $\hat{a}_1$ and 23 percent of the $\hat{a}_2$ insignificant.

This appears to indicate that two security prices, however they are selected, are not sufficient to explain a third security price. This argument is reinforced by relatively low adjusted R-squares. The average adjusted R-square value was .421, ranging from negatives to the highest .804. For the ten regressions in which both coefficients were statistically significant, the adjusted R-squares ranged from

TABLE 2

Statistics From Regressions: Three Prices

$$P^*_{it} = a_0 + a_1 P^*_{jt} + a_2 P^*_{kt} + e_{it},\ i = 1, \ldots, 10$$

Regressant Portfolio	Regressor Portfolio	$t(\hat{a}_0)$	$t(\hat{a}_1)$	$t(\hat{a}_2)$	$\bar{R}^2$	F^a	D-W Statistic	$\hat{\rho}(t)^b$
1	2, 9	-.64	5.00	1.08	.569	16.19	1.36	-.27(-1.37)
1	3, 8	-.03	3.97	.56	.397	8.57	1.99	.07(.39)
1	4, 7	-.52	2.47	3.37	.628	20.41	1.94	.00(.04)
1	5, 6	-.45	4.86	.95	.496	12.35	2.22	.12(.63)
2	1, 10	1.66	6.17	2.44	.646	21.99	1.00	-.31(-1.59)
2	3, 8	1.76	8.99	2.59	.804	48.40	1.85	-.03(-.17)
2	4, 7	.57	2.63	2.39	.549	15.03	1.05	-.24(-1.25)
2	5, 6	.76	5.77	3.20	.647	22.09	1.42	-.19(-.96)
3	1, 10	.23	4.20	1.15	.424	9.48	1.97	.15(.74)
3	2, 9	-1.69	8.92	-1.70	.773	40.33	2.32	.19(.96)
3	4, 7	-.76	3.51	.99	.512	13.09	1.76	.12(.62)
3	5, 6	-.45	5.04	4.30	.649	22.32	1.91	.04(.24)
4	1, 10	1.44	4.43	.75	.441	10.10	2.06	.04(.20)
4	2, 9	.60	4.14	.10	.427	9.57	1.71	-.08(-0.40)
4	3, 8	1.37	4.81	.36	.492	12.17	1.83	.05(.26)
4	5, 6	1.03	5.74	1.28	.585	17.22	2.56	.28(1.46)
5	1, 10	.80	5.02	1.47	.523	13.65	1.93	.04(.20)
5	2, 9	.14	4.34	1.53	.527	13.85	1.53	-.08(-.44)
5	3, 8	.50	3.61	.71	.356	7.35	1.68	.08(.40)
5	4, 7	.10	3.85	5.06	.802	47.58	2.89	.47(2.61)

a $F_{2,\ 21,\ .05} = 3.47$ and $F_{2,\ 21,\ .01} = 5.78$

b $\hat{\rho}$ is the first-order autoregressive coefficient or residuals.

TABLE 2 (continued)

Statistics From Regressions: Three Prices

$$P^*_{it} = a_0 + a_1 P^*_{jt} + a_2 P^*_{kt} + e_{it},\ i = 1, \ldots, 10$$

Regressant Portfolio	Regressor Portfolios	$t(\hat{a}_0)$	$t(\hat{a}_1)$	$t(\hat{a}_2)$	$\bar{R}^2$	F^a	D-W Statistic	$\hat{\rho}(t)^b$
6	1, 10	.14	.70	.14	-.068	.25	1.73	.03(.16)
6	2, 9	-.55	2.47	-1.44	.169	3.33	1.91	.02(.11)
6	3, 8	.09	3.01	-.52	.235	4.53	1.72	-.03(-.18)
6	4, 7	-.10	.85	-.40	-.057	.37	1.87	.09(.46)
7	1, 10	.21	6.30	3.14	.673	24.71	1.67	-.07(-.35)
7	2, 9	-.16	4.03	3.84	.648	22.20	1.60	-.12(-.63)
7	3, 8	-.13	2.85	1.71	.320	6.43	1.53	-.01(-.05)
7	5, 6	-.79	6.85	.09	.661	23.50	2.64	.32(1.70)
8	1, 10	.55	1.37	5.27	.545	14.80	2.01	.10(.50)
8	2, 9	.10	1.23	2.31	.245	4.75	1.91	.06(.34)
8	4, 7	.03	-.32	1.69	.061	1.75	1.98	.06(.37)
8	5, 6	-.17	1.00	-.07	-.044	.50	2.05	.08(.43)
9	1, 10	-.65	2.64	4.70	.540	14.51	2.23	.31(1.62)
9	3, 8	-.90	.13	2.67	.191	3.72	1.89	.16(.81)
9	4, 7	-.67	-1.21	4.06	.416	9.22	2.12	.11(.59)
9	5, 6	-1.09	2.20	-.74	.127	2.67	2.12	.10(.52)
10	2, 9	-.22	.65	3.71	.399	8.64	2.36	.23(1.18)
10	3, 8	-.81	.47	4.94	.509	12.96	2.22	.17(.85)
10	4, 7	-.42	-.61	1.86	.072	1.90	2.24	.13(.66)
10	5, 6	-.70	.97	.12	-.046	.48	2.25	.13(.65)

a $F_{2,\ 21,\ .05} = 3.47$ and $F_{2,\ 21,\ .01} = 5.78$

b $\hat{\rho}$ is the first-order autoregressive coefficient of residuals.

.540 to .804. Therefore, Hypothesis 3 does not seem to be supported. The first-order autoregressive parameters of residuals had absolute values of the t-statistics smaller than 2 in all regressions.

The other hypotheses, 4 and 5, concern whether there are any systematic changes in either the intercept or the adjusted R-square as the number of independent variables in equation (40) increases. In order to test these propositions, the following regression was run for each of ten portfolios:

$$p_{it} = a_0 + \sum_{\substack{j=1 \\ j \neq 1}}^{10} a_j p^*_{jt} + e_{it}, \quad i = 1,\ldots,10 \tag{41}$$

where $p^*_{it} = p_{it} - p_{i,t-1}$ and $e_{it} = \rho e_{i,t-1} + u_{it}$. The results are presented in Table 3. The intercept never turns out to be significant. This can be said to support Hypothesis 4. The adjusted R-squares are generally quite high except .185 for portfolio 6.

F'-statistics of Table 3 need to be explained. They are F-statistics pertinent for testing the null hypothesis that seven extra variables added to each of the previous regressions in Table 2 do not have additional explanatory power as a group. Therefore, there should be four F'-statistics corresponding to four regressions for each dependent-variable portfolio in Table 2. Numbers in the fourth column of Table 3 indicate the frequency with which F'-statistics is significant at least at the 5 percent level for each portfolio. For example, the first number 4 means that F'-statistics is as many as four times (100 percent) statistically significant for portfolio 1 as the dependent variable. The results show that there are predominantly significant changes in the adjusted R-square values.

TABLE 3

Statistics From the Regressions: All Prices

$$p^*_{it} = a_0 + \sum_{\substack{j=1 \\ j \neq i}}^{10} a_j p^*_{jt} + e_{it}, \quad i = 1, \ldots, 10$$

Regressant portfolio	$t(\hat{a}_0)$	$\bar{R}^2$	F^a	# of times Sign. F'^b	D-W Statistic	$\hat{\rho}(t)^c$
1	-.92	.773	9.73**	4	1.57	-.20(-1.02)
2	1.81	.820	12.70**	3	1.18	-.39(-2.08)
3	-1.51	.821	12.76**	3	1.99	.06(.30)
4	1.40	.604	4.90**	0	2.38	.20(1.04)
5	-.55	.767	9.45**	3	3.05	.53(3.13)
6	.14	.185	1.58	0	1.55	-.21(-1.08)
7	.52	.768	9.47**	1	2.82	.42(2.27)
8	-.20	.479	3.35*	2	1.50	-.16(-.80)
9	-.37	.648	5.70**	3	2.65	.36(1.89)
10	-.32	.750	8.69**	4	2.23	.14(.72)

[a] F is pertinent for testing the null hypothesis that a_j's, $j \neq i$, are jointly zero.

[b] F' is to test the influence of additional explanatory variables which are not included in each of the previous regressions. Numbers indicate the cases in which F' is significant at least at the 5 percent level.

[c] $\hat{\rho}$ is the first-order autoregressive coefficient of residuals.

* denotes significant at the 5 percent level.
** denotes significant at the 1 percent level.

In order to test whether dividend has additional explanatory power for a security price, the following regression was run:

$$p^{*}_{it} = a_0 + \sum_{\substack{j=1 \\ j\neq i}}^{10} a_j p^{*}_{jt} + \sum_{k=1}^{10} b_k d^{*}_{kt} + e_{it}, \quad i = 1,\ldots,10 \qquad (42)$$

where $p^{*}_{jt} = p_{jt} - p_{j,t-1}$, $d^{*}_{kt} = E_t d_{k,t+1} - E_{t-1} d_{k,t+1}$, and $e_{it} = \rho e_{i,t-1} + u_t$. F-statistics (F' in the table) pertinent for testing that b_k's are jointly zero are reported in Table 4. They are all significant except for portfolios 4 and 6. The adjusted R-square values increase substantially over 95 percent with two exceptions (.758 for portfolio 4 and .711 for portfolio 6). These results appear to support the model (42).

The results reported up to this point clearly provide evidence for the model. Cheng and Grauer rejected the SLM model because their regression results contradicted the hypotheses based on their three-security model. However, if Cheng and Grauer's model may be interpreted as a two-factor arbitrage pricing model then it implies that two factors account for all variances in security prices. This is too strong to serve as a basis for rejecting the capital asset pricing model. In fact, various studies found that more than two factors were needed to explain security returns (for example, Roll and Ross (1980) and Sharpe (1982)). Hence, Cheng and Grauer's model appears to be misspecified with respect to equations (41) and (42). If either of the two is the correct model then their three-security model may well be rejected by the theory and empirical results derived in this paper.

TABLE 4

Summary Statistics From the Regressions: Prices and Dividends

$$p^*_{it} = a_0 + \sum_{\substack{j=1 \\ j\neq 1}}^{10} a_j\, p^*_{jt} + \sum_{k=1}^{10} b_k d^*_{kt} + e_{it},\ i = 1, \ldots, 10$$

Regressant Portfolio	$\bar{R}^2$	F^a	F'^b	D-W Statistic	$\hat{\rho}(t)^c$
1	.996	393.17**	102.54**	1.58	-.20(-1.02)
2	.975	48.93**	9.78*	1.87	-.04(-.23)
3	.997	587.52**	119.88**	1.66	-.16(-.80)
4	.758	4.80	1.89	1.60	-.19(-.98)
5	.994	232.49**	62.05**	1.70	-.14(-.71)
6	.711	3.97	3.54	1.53	-.21(-1.10)
7	.991	146.82**	38.98**	1.66	-.16(-.81)
8	.956	27.30**	16.14**	1.42	-.28(-1.46)
9	.990	127.25**	51.45**	1.63	-.17(-.86)
10	.995	302.00**	86.74**	1.56	-.21(-1.08)

[a] F is pertinent for testing the null hypothesis that a_j's, $j \neq i$, and b_k's are jointly zero.

[b] F' is to test the influence of additional explanatory variables d_{kt}, $k = 1, \ldots, 10$, as a group.

[c] $\hat{\rho}$ is the first-order autoregressive coefficient of residuals.

* denotes significant at the 5 percent level.
** denotes significant at the 1 percent level.

D. A Structural-Form Model to Test the Supply Effect

D.1 Identification of Structural-Form Equations

In the previous section, the model was tested using the OLS mainly to show that a security price is determined not only by two other randomly selected security prices but by all other prices and dividends. If a simultaneous equations system is exactly identified there is one-to-one correspondence between structural-form parameters and reduced-form parameters. If a system is overidentified there are too many restrictions to estimate parameters. If underidentified, then the parameters cannot be uniquely estimated. This section considers the identification problem of the model defined in equations (23) and (24). In the next section, the supply effect will be formally tested by estimating the reduced-form equations rather than the structural-form equations.

The equation to be identified and estimated is (23). Since IMA(0,1,0) process seemed to fit price series, equation (23) was reduced to equation (43):

$$(G_0 + W_0)(p_t - p_{t-1}) + \Gamma_0(E_t d_{t+1} - E_{t-1} d_{t+1}) = u_t. \tag{43}$$

Define $p_t - p_{t-1} = p_t^*$ and $E_t d_{t+1} - E_{t-1} d_{t+1} = d_t^*$. Equation (43) becomes

$$(G_0 + W_0) p_t^* + \Gamma_0 d_t^* = u_t, \tag{44}$$

or

$$G p_t^* + H_t d^* = u_t, \tag{45}$$

where $G = G_0 + W_0$ and $H = \Gamma_0$. By definitions of coefficients in equation (23),

$$G = \begin{bmatrix} g_{11} g_{12} & \cdots & g_{1n} \\ g_{21} g_{22} & \cdots & g_{2n} \\ \cdot \quad \cdot & \cdots & \cdot \\ \cdot \quad \cdot & \cdots & \cdot \\ g_{n1} g_{n2} & \cdots & g_{nn} \end{bmatrix} = - \begin{bmatrix} r^*s_{11}+a_1b_1 & r^*s_{12} & \cdots & r^*s_{1n} \\ r^*s_{21} & r^*s_{22}+a_2b_2 & \cdots & r^*s_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ r^*s_{n1} & r^*s_{n2} & \cdots & r^*s_{nn}+a_nb_n \end{bmatrix}$$

$$H = \begin{bmatrix} h_{11} h_{12} & \cdots & h_{1n} \\ h_{21} h_{22} & \cdots & h_{2n} \\ \cdot \quad \cdot & \cdots & \cdot \\ \cdot \quad \cdot & \cdots & \cdot \\ h_{n1} h_{n2} & \cdots & h_{nn} \end{bmatrix} \begin{bmatrix} s_{11}+a_1 & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22}+a_2 & \cdots & s_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ s_{n1} & s_{n2} & \cdots & s_{nn}+a_n \end{bmatrix} . \quad (46)$$

If the matrix G is assumed to be nonsingular the reduced-form of the model may be written:

$$p_t^* = \Pi d_t^* + v_t, \quad (47)$$

where Π is a n x n matrix of the reduced-form coefficients and v_t is a column vector of n reduced-form disturbances or

$$\Pi = -G^{-1}H \quad (48)$$

and

$$v_t = G^{-1}u_t. \tag{49}$$

The first step in an econometric investigation of the model is the specification of the system (45). Without a priori knowledge of the system, all equations of the model would look alike statistically in which each equation is a linear combination of all endogenous and all exogenous variables. No equation contains any single variable which does not appear in any other equations. Observational information can only distinguish the true equation from candidates which are not linear combinations of the equations of the model. This means that there are infinite number of structural forms corresponding to the same reduced-form: the structural parameters cannot be uniquely determined with respect to the reduced-form parameters.

The specification will rely heavily on accepted financial theory or on a priori knowledge of the simultaneous system. This knowledge may be in the form of restrictions on the structural parameters. For example, if certain variables are known not to play any direct roles in a particular equation certain elements in the rows of the matrices G and H corresponding to that equation will be zero (exclusion restrictions). Restrictions may also be placed on combinations of elements in the two matrices.[8]

Following Johnson (1972), equation (48) can be written as:

$$AW = 0, \tag{50}$$

where

$$A = [G\ H] \text{ and } W = [\Pi\ I_n]'.$$

A is the n x (2n) matrix of all structural coefficients in the model and W is a matrix of order (2n) x n, which has rank n. The first equation in (50) may be expressed as:

$$A_1 W = 0, \tag{51}$$

where A_1 is the first row of A. Since the elements of Π can be consistently estimated and I_n is the identity matrix of order n equation (51) contains 2n unknowns (n in the first row of G and n in the first row of H) in terms of n π's. Therefore, 2n unknowns cannot be determined from this equation alone. However, if there are n-1 restrictions on the parameters, (51) will be solved uniquely, with a normalization rule.

The rank condition (necessary and sufficient) for the identifiability of the first equation of the model is:

$$\rho[W\ \phi] = n + n - 1, \tag{52}$$

where ϕ has 2n rows and a column for each restriction. Johnston further shows that:[9]

$$\rho[W\ \phi] = 2n - 1 \quad \text{iff} \quad \rho(A\ \phi) = n - 1. \tag{53}$$

The order condition (necessary but not sufficient) for the first equation is

$$\rho(\phi) \geq n - 1. \tag{54}$$

In other words, there must be at least n - 1 independent restrictions.[10]

It can be illustrated, with three endogenous and three exogenous variables, that the system (45) is exactly identified although the case of n = 10 is entirely similar. That is, the model can be expressed in the form:

$$-\begin{bmatrix} r^*s_{11}+a_1b_1 & r^*s_{12} & r^*s_{13} \\ r^*s_{21} & r^*s_{22}+a_2b_2 & r^*s_{23} \\ r^*s_{31} & r^*s_{32} & r^*s_{33}+a_3b_3 \end{bmatrix} \begin{bmatrix} p^*_{1t} \\ p^*_{2t} \\ p^*_{3t} \end{bmatrix}$$

$$+\begin{bmatrix} s_{11}+a_1 & s_{12} & s_{13} \\ s_{21} & s_{22}+a_2 & s_{23} \\ s_{31} & s_{32} & s_{33}+a_3 \end{bmatrix} \begin{bmatrix} d^*_{1t} \\ d^*_{2t} \\ d^*_{3t} \end{bmatrix} = \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}, \qquad (55)$$

where r^* = scalar risk-free rate, s_{ij} = elements of variance-covariance matrix of return, a_i = supply adjustment cost of firm i, b_i = overall cost of capital of firm i.

The prior restrictions on the first equation take the form, using the notations in (46)

$$g_{12} = -r^*h_{12},\ g_{13} = -r^*h_{13},$$

$$g_{21} = -r^*h_{21},\ g_{23} = -r^*h_{23},$$

$$g_{31} = -r^*h_{31},\ g_{32} = -r^*h_{32},$$

Thus for the first equation, the restriction matrix may be written as

$$\phi = \begin{bmatrix} 0 & 1 & 0 & 0 & r^* & 0 \\ 0 & 0 & 1 & 0 & 0 & r^* \end{bmatrix}', \quad A\phi = \begin{bmatrix} 0 & 0 \\ -a_2(b-r^*) & 0 \\ 0 & -a_3(b-r^*) \end{bmatrix}.$$

The rank of ϕ is 2, and since $n = 3$ in this example, the order condition is obviously satisfied. $A\phi$ is the rank condition of A. Thus $\rho(A\phi) = 2 = n - 1$ and the first equation is identified, provided that $b_2 - r^* \neq 0$ and $b_3 - r^* \neq 0$. Theoretically and empirically, these requirements are satisfied since overall cost of a firm should be higher than the risk-free rate.

Alternatively, the relations from equation (51) in the parameters of the first equation give

$$[g_{11} g_{12} g_{13} h_{11} h_{12} h_{13}] \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{13} & 0 & 0 \\ \pi_{21} & \pi_{22} & \pi_{23} & 1 & 0 \\ \pi_{31} & \pi_{32} & \pi_{33} & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & r^* & 0 \\ 0 & 0 & 1 & 0 & r^* \end{bmatrix} = [0 \quad 0 \quad 0 \quad 0 \quad 0]$$

that is,

$$g_{11}\pi_{11} + g_{12}\pi_{21} + g_{13}\pi_{31} + h_{11} = 0, \; g_{11}\pi_{12} + g_{12}\pi_{22} + g_{13}\pi_{32} + h_{12} = 0,$$

$$g_{11}\pi_{13} + g_{12}\pi_{23} + g_{13}\pi_{33} + h_{13} = 0, \; g_{12} + r^*h_{12} = 0, \; g_{13} + r^*h_{13} = 0.$$

Imposing normalization rule by setting $g_{11} = 1$, these equations can be solved for the three unknowns g_{12}, g_{13}, h_{11}, in terms of π's. This shows explicitly that the parameters of the first equation may be derived uniquely from those of the reduced form. Thus the first equation is exactly identified. In the similar fashion, it can be shown that the second and third equations are also exactly identified.[11]

D.2 Test of Supply Effect

In order to test supply effect, the reduced-form equations may be used since the structural-form equations are exactly identified.[12] Equation (47) can be rewritten as, for the case of two portfolios,

$$G = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} -(r^*s_{11}+a_1b_1) & -r^*s_{12} \\ -r^*s_{21} & -(r^*s_{22}+a_2b_2) \end{bmatrix}$$

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} s_{11}+a_1 & s_{12} \\ s_{21} & s_{22}+a_2 \end{bmatrix}.$$

Thus,

$$-G^{-1}H = \begin{bmatrix} r^*s_{11}+a_1b_1 & r^*s_{12} \\ r^*s_{21} & r^*s_{22}+a_2b_2 \end{bmatrix}^{-1} \begin{bmatrix} s_{11}+a_1 & s_{12} \\ s_{21} & s_{22}+a_2 \end{bmatrix}$$

$$= \frac{1}{|G|} \begin{bmatrix} r^*s_{22}+a_2b_2 & -r^*s_{12} \\ -r^*s_{21} & -r^*s_{11}+a_1b_1 \end{bmatrix} \begin{bmatrix} s_{11}+a_1 & s_{12} \\ s_{21} & s_{22}+a_2 \end{bmatrix}$$

$$= \frac{1}{|G|}\begin{bmatrix} (r^*s_{22}+a_2b_2)(s_{11}+a_1) - r^*s_{12}s_{21} & (r^*s_{22}+a_2b_2)s_{12} - r^*s_{12}(s_{22}+a_2) \\ -rs_{21}(s_{11}+a_1) + (r^*s_{11} + a_1b_1)s_{21} & -r^*s_{21}s_{12} + (r^*s_{11}+a_1b_1)(s_{22}+a_2) \end{bmatrix}$$

$$= \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix}.$$

If $a_1 = a_2 =$ then, with $s_{12} = s_{21}$, the matrix would become a scalar matrix in which diagonal elements are equal to $r^*(s_{11}s_{22}-s_{12}^2)$ and off-diagonal elements are all zero. Three-equation case is shown in Appendix A. In a similar fashion, this can be generalized into the case of ten portfolios.

Based on the results above, two hypotheses are to be tested about the parameters in the equation (47):

$$p_t^* = \Pi d_t^* + v_t.$$

Hypothesis 1: All the off-diagonal elements in the coefficient matrix Π are zero if the supply effect does not exist; or equivalently, the price change of a security is determined by only its own dividend term if the supply effect does not exist.

Hypothesis 2: All the diagonal elements in the coefficient matrix Π are equal in the magnitude if the supply effect does not exist; or equivalently, price changes of each security are related to its own dividends in the same fashion, proportional to level of dividends.

These two hypotheses should be satisfied jointly. In other words, if supply effect does not exist price changes of a security should be a function of its own dividend expectation adjustments and the coefficients should be all equal across securities. In the traditional financial theory, a security price is defined as the discounted value of the entire dividend stream in the future. The undiscounted dividend stream would basically determine the relative magnitude of the security price and the discounting process--mainly the investors. Consensus about the security's risk would determine the relative quality of the dividend stream.

In the model, if there were any change in the security price it should be the result of change in the prediction of the next dividend due to the additional information during the current period. This adjustment may result in greater or smaller changes in prices of different securities due to the absolute changes in the expectation error. However, the way in which the expectation errors in dividends are built in the current price is the same for all securities. This would happen if supply of securities is fixed and thus price changes would be influenced by only its own dividend expectation errors. If supply of securities is flexible responding to changes in demand by investors possessing now modified expectations about future dividends of all securities, then the change in a security price would be influenced by expectation adjustments in dividends of all other securities as well as that in its own dividend.

In order to test Hypothesis 1, two regressions were run for each dependent-variable portfolio:

$$p^*_{it} = a_i + b_i d^*_{it} + e_{it}, \quad i = 1,\ldots,10$$

$$p^*_{it} = a'_i + b'_i d^*_{it} + \sum_{j \neq i} b'_j d^*_{jt} + e'_{it}, \quad i = 1,\ldots,10 \qquad (56)$$

where $p^*_{it} = p_{it} - p_{i,t-1}$ and $d^*_{it} = E_t d_{i,t+1} - E_{t-1} d_{i,t+1}$. The null hypothesis states that b_j's, $j \neq i$, are all zero in the second regression of (56). The results of regressions are presented in Table 5. The table contains ten pairs of regressions: regression of a price variable on its own dividend variable only is shown in the first row and regression of a price variable on all dividend variables in the second row. F-statistics are pertinent for testing the significance of each regression as a whole whether there is only one or more than one independent variable. F'-statistics are pertinent for testing the influence of the extended set of explanatory variables:

$$\frac{(SSR_Q - SSR_K)/(Q - K)}{SSE_Q/(n - Q)}$$

where SSR_Q = sum-of-squares regression with all dividend terms as independent variables with $Q = 11$ degrees of freedom,

SSR_K = sum-of-squares regression with diagonal dividend only with $K = 1$ degrees of freedom,

SSE_Q = sum-of-squares error with all dividend terms as independent variables with Q degrees of freedom,

$n = 24$ = number of observations.

TABLE 5

Test of Supply Effect on Off-Diagonal Terms:

$$P^*_{it} = a_i + b_i d^*_{it} + e_{it}, \quad i = 1, \ldots, 10$$

$$P^*_{it} = a_i' + b_i' d^*_{it} + \sum_{j \neq i} b_j' d^*_{jt} + e_{it}', \quad i = 1, \ldots, 10$$

Dependent Variable	Independent Variables	R^2	$\bar{R}^2$	F^a	F'^b	D-W Statistics
P_1	D_1	.043	-.000	.99		1.74
P_1	D_1-D_{10}	.675	.425	2.70**	2.80**	1.76
P_2	D_2	.006	-.039	.13		1.71
P_2	D_1-D_{10}	.717	.499	3.29**	3.63***	1.99
P_3	D_3	.048	.004	1.11		1.94
P_3	D_1-S_{10}	.627	.341	2.19*	2.25*	2.06
P_4	D_4	.242	.207	7.03**		1.18
P_4	D_1-D_{10}	.485	.090	1.22	.68	1.43
P_5	D_5	.063	.021	1.49		1.55
P_5	D_1-D_{10}	.671	.418	2.65**	2.67*	1.25
P_6	D_6	.154	.116	4.03**		2.03
P_6	D_1-D_{10}	.438	.006	1.01	.73	2.11
P_7	D_7	.008	-.036	.18		2.09
P_7	D_1-D_{10}	.540	.186	1.52	1.67	1.54
P_8	D_8	.030	-.013	.70		2.21
P_8	D_1-D_{10}	.765	.585	4.25***	4.53***	1.98
P_9	D_9	.060	.018	1.42		2.28
P_9	D_1-D_{10}	.554	.211	1.61	1.59	2.32
P_{10}	D_{10}	.004	-.040	.108		2.52
P_{10}	D_1-D_{10}	.556	.215	1.63	1.80	2.07

[a] F is pertinent for testing the null hypothesis that all independent variables are jointly zero in each regression.

[b] F′ is pertinent for testing the null hypothesis that b_j's are jointly zero.

* denotes significant at the 10 percent level.
** denotes significant at the 5 percent level.
*** denotes signidicant at the 1 percent level.

The null hypothesis is rejected at different levels of significance in five out of ten portfolios. This evidence seems to be insufficient to reject the null hypothesis universally.

To test Hypothesis 2, two estimation methods were employed: covariance analysis and seemingly unrelated regressions. There are basically four single-equation estimation schemes by which time-series and cross-section data might be pooled. The first technique is simply to combine all the data and perform the ordinary least squares regression on the entire data set. The second approach recognizes that omitted variable may lead to changing cross-section and time-series intercepts and/or slopes. Covariance analysis involves the addition of dummy variables to the model to allow for these changing intercepts and/or slopes.[13] The third procedure improves the efficiency of the OLS method by accounting for existence of cross-section and time-series disturbances. The error component procedure is an application of the generalized least squares estimation process. The last technique accounts for the fact that the error term may be correlated over time and over cross-section units. The second technique--regression with dummy variables will be first used to test Hypothesis 2.

All the estimation procedures described to this point are single-equation methods which do not account for the fact that error terms across equations for corresponding observations may be correlated. In other words, each single-equation method yields inefficient estimates of parameters because all the information available in the description of the system of equations is not used in the estimation procedure. One system estimation method--seemingly unrelated regressions technique will be further adopted to test Hypothesis 2.

The first estimation involves pooling of time-series and cross-section with dummy variables:[14]

$$p_t^* = \alpha + \beta d_t^* + \sum_{i=2}^{10} \gamma_i d_t^* z_{it} + \varepsilon_t, \tag{57}$$

where z_{it} are used for portfolio identification as follows:

$$z_{it} = 1 \text{ for portfolio } i,$$
$$= 0 \text{ otherwise.}$$

The null hypothesis is

$$\gamma_2 = \gamma_3 = \ldots = \gamma_{10} = 0$$

against the alternative hypothesis that the null hypothesis is not true; that is, γ's are not jointly zero.

Table 6 presents the results. The first regression was a simple application of the OLS to the pooled data. F-statistics is significant at the 1 percent level. The second regression included 9 slope dummy variables to test any significant differences among the slope coefficients. F'-statistic is pertinent for testing that all γ's in equation (48) are jointly zero. It appears to be insignificant even at the 10 percent level.

There are two problems associated with the use of the regressions with dummy variables method. First, coefficients of dummy variables are hard to interpret since the method does not attempt to identify the variables underlying the change. Second, the method uses up a substantial number of degrees of freedom (10 - 1 = 9 in the model) and thus may suffer from decrease in its statistical power.

TABLE 6

Test of Supply Effect on Diagonal Terms:
Regression with Dummy Variables

$$p_t^* = \alpha + \beta d_t^* + \sum_{i=2}^{10} \gamma_i d_t^* z_{it} + \varepsilon_t$$

	R	R	F	F'	D-W
Without Dummy Variables	.025	.021	6.82**		2.00
With Dummy Variables	.076	.036	1.89*	1.39	1.94

F' is pertinent for testing that all γ's are jointly zero.

* denotes significant at the 5 percent level.

** denotes significant at the 1 percent level.

The second estimation to test supply effect on diagonal terms is the seemingly unrelated regressions model which consists of a series of equations linked because the error terms across equations are correlated. The primary link is caused, as the model (48) specified, the spill-over effect among security prices. This effect should be reflected in the error terms. Seemingly unrelated regressions method, credited to Zellner (1962), yields the generalized least squares estimator that is asymptotically more efficient than the ordinary least squares estimator obtained from a single-equation system.

The system of which (47) is an equation may be written as, if π is a diagonal:

$$p_i^* = d_i^* \pi_i + v_i \quad (i=1,2,\ldots,n) \tag{58}$$

or

$$p_m^* = d_m^* \pi_m + v_m \tag{59}$$

where p_m^* is a (Tx1) vector of observations of the dependent variable, d_m^* is a (Tx1) vector of observations of the independent variable, π_m is a scalar regression coefficient, and v_m is a (Tx1) vector of error terms. The system of equations (58) can be represented as:

$$\begin{bmatrix} p_1^* \\ p_2^* \\ \cdot \\ \cdot \\ p_n^* \end{bmatrix} = \begin{bmatrix} d_1^* & 0 & 0 & \cdots & 0 \\ 0 & d_2^* & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & d_n^* \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \cdot \\ \cdot \\ \pi_n \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_n \end{bmatrix} \tag{60}$$

or more compactly as:

$$p = D\pi + v, \tag{61}$$

where p = a (nTx1) vector, D is a (nTxn) block-diagonal matrix, π is a (nx1) vector, and the dimension of v is (nTx1). The random error vector v in system (60) and (61) is assumed to have the following variance-covariance matrix:

$$\Omega = E(v\ v') = \begin{bmatrix} \sigma_{11}I & \sigma_{12}I & \cdots & \sigma_{1n}I \\ \sigma_{21}I & \sigma_{22}I & \cdots & \sigma_{2n}I \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \sigma_{n1}I & \sigma_{12}I & \cdots & \sigma_{nn}I \end{bmatrix} \tag{62}$$

where I is an identity matrix of order TxT. The information about the correlation of the error terms across equations is then contained in the description of the matrix Ω.

The best linear unbiased estimator of π is given by Aitken's generalized least squares formula as:

$$\pi^* = (D'\Omega^{-1}D)^{-1}(D'\Omega^{-1}p) \tag{63}$$

However, if Ω is unknown it is impossible to use (63) in practice. Zellner suggested an estimate using variances and covariances from the ordinary least squares residuals:

$$(T-k)\hat{\Omega} = (T-k)(s_{ij}) = (v_i'v_j)$$

$$= ((p_i - D_i\hat{\pi}_i)'(p_j - D_j\hat{\pi}_j))$$

where $\hat{\pi}_i$ is the usual single-equation OLS estimator. The resulting estimator of

$$\pi^{**} = (D'\hat{\Omega}^{-1}D)^{-1}(D'\hat{\Omega}^{-1}p) \tag{64}$$

is called a two-stage Aitken estimator.

Zellner further proposed a procedure to test the equality of all regression coefficients:

$$\pi_1 = \pi_2 = \dots = \pi_n. \tag{65}$$

The hypothesis in (65) states that all cross-section units are homogeneous insofar as their regression coefficients are concerned.[15] This is exactly what was intended to test in Hypothesis 2. The test statistic is given as:

$$F_{q,n(T-1)} = \frac{p'\Omega^{-1}D(D'\Omega^{-1}D)^{-1}C'(C(D'\Omega^{-1}D)^{-1}C')^{-1}C(D'\Omega^{-1}D)^{-1}D'\Omega^{-1}p}{p'\Omega^{-1}p - p'\Omega^{-1}D(D'\Omega^{-1}D)^{-1}D'\Omega^{-1}p}$$

$$\times\ n\ (T-k)/q \qquad (66)$$

where (nxT) is number of observations, (nxk) the number of independent variables, q number of restrictions on the system. The restrictions given in the hypothesis (65) can be expressed as follows:

$$C\pi = \begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & -1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \begin{bmatrix} \pi_1 \\ \pi_2 \\ \cdot \\ \cdot \\ \pi_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \qquad (67)$$

Results of comparing the OLS and the SUR estimates are shown in Table 7.[16] It is seen that application of the SUR has resulted in a significant reduction in the estimated coefficient estimator variances as compared with those of the OLS. t-statistics for the slope coefficient from the OLS was significant at the 1 percent level for portfolio 4 and at the 5 percent level for portfolio 6. This was previously shown by F-statistics in Table 5. The slope coefficient was significant at the 1 percent level for portfolios 1, 3, 4, 6, 8, and 9, and at the 5 percent level for portfolio 10. F-statistic given in (66) was:

$$F_{9,10(24-2)} = \frac{18.0725}{1.0431} \times \frac{9}{10(24-2)}$$

$$= 17.325$$

TABLE 7

Test of Supply Effect on Diagonal Terms: The OLS vs. the SUR

Dependent Variable	Estimation Method	Intercept (Standard Error)	Slope (Standard Error)
P_1	OLS	-.1134(.7112)	10.5541(10.5069)
	SUR	-.1508(.7028)	14.0065(3.0583)**
P_2	OLS	.0797(.6514)	-1.0800(9.0784)
	SUR	.0065(.6408)	3.9669(4.2000)
P_3	OLS	-.0377(.5984)	12.4873(11.8249)
	SUR	-.0375(.5938)	12.4600(2.9033)**
P_4	OLS	.4859(.4484)	-29.1381(9.1827)**
	SUR	.4679(.4449)	-26.3863(3.1318)**
P_5	OLS	.6424(.5952)	-35.2258(28.7702)
	SUR	.2540(.5135)	-3.4127(14.8340)
P_6	OLS	-.2588(.6687)	10.3497(4.5027)*
	SUR	-.2582(.6684)	10.4526(2.5843)**
P_7	OLS	.0357(.5237)	-4.3749(10.2957)
	SUR	.0033(.4911)	-2.6916(4.1103)
P_8	OLS	-.3935(.5707)	10.3082(12.3102)
	SUR	-.5878(.4226)	15.8729(5.5599)**
P_9	OLS	-.4822(.5179)	4.0842(3.4225)
	SUR	-.4828(.5173)	4.1581(1.1084)**
P_{10}	OLS	-.1722(.2528)	7.1103(21.5141)
	SUR	-.1908(.2441)	13.0010(5.3499)*

* denotes significant at the 5 percent level.

** denotes significant at the 1 percent level.

which can be compared with $F_{9,220,.01} = 2.50$ ($P(F > 17.325) = .0001$). This leads to rejection, at the 1 percent level, of the null hypothesis that the slope coefficients are equal across all equations and therefore, it can be refuted that supply effect does not exist.

In conclusion, the test results presented in Tables 5 through 7 are sufficient to reject the two null hypotheses of non-existence of supply effect in capital asset pricing.

E. Summary

In this paper, following Black (1976) a dynamic capital asset pricing model in terms of rational expectations is theoretically derived. This new theoretical model is compared with previous capital asset pricing models. It is shown that the theoretical asset pricing model derived in this paper is a generalized case of Cheng and Grauer's (1980) model and Cheng and Grauer's model might be misspecified.

Using portfolio data, the new model derived in this paper is empirically tested. It is found that SLM's type of CAPM might be held if the model is correctly specified. In addition, we also theoretically and empirically show that there exist some supply effects in capital asset pricing determination processes as theoretically shown by Senbet and Taggart (1984).

Footnotes

1. It is assumed that the dividend or coupon is paid at the end of each period. This may be a little different from the usual treatment of dividend in the calculation of holding period return. The numerator in the holding period return formula is $p_{t+1} - p_t + d_{t+1}$ where d_{t+1} is the dividend received "during" period t.

2. $$V(W_{t+1}) = E(W_{t+1} - \overline{W}_{t+1})(W_{t+1} - \overline{W}_{t+1})'$$

$$= E(q_t'x_t - q_t'\overline{x}_t)(q_t'x_t - q_t'\overline{x}_t)$$

$$= Eq_t'(x_t - \overline{x}_t)(x_t - \overline{x}_t)'q_t$$

$$= q_t'Sq_t, \text{ where } S = E(x_t - \overline{x}_t)(x_t - \overline{x}_t)'$$

3. From equation (12) we have, by setting $Q = \overline{Q}$ which is the fixed supply of securities in the standard CAPM,

$$\overline{Q}_t = cS^{-1}(\overline{x}_t - r^*p_t)$$

$$c^{-1}S\overline{Q}_t = \overline{x}_t - r^*p_t$$

$$p_t = 1/r^*(\overline{x}_t - c^{-1}S\overline{Q}_t)$$

where c^{-1} is the market price of risk. Notice its similarity to the original Lintner's equation (1965).

4. Senbet and Taggart (1984) show the impacts of changes in the firms' capital structure on investors' wealth. Starting with the standard investors' maximization of expected utility of current and future consumption under the different lending and borrowing rates, they show that investors who borrow on their own accounts would like firms in which they hold shares to borrow more and investors who are lenders would prefer more lending by firms. However, these conflicts can be balanced by firms' adjustments of their capital structures to serve the needs of different clienteles. They argue that the standard CAPM relationship can be restored through this intermediation by firms if the supply adjustments are costless.

5. The existence of feedback between dividends and earnings series was empirically tested. It is found there is no feedback between these two series.

6. For each of the three dependent-variable portfolios, there are 171 combinations of two independent variables from nineteen candidate variables. That is:

$$\binom{19}{2} = \frac{19!}{(2!)(17!)} = 171$$

Therefore, nine regressions are only one nineteenth of the total number of combinations.

7. For each of ten portfolios, there are 36 combinations of two independent variables. That is,

$$\binom{9}{2} = \frac{9!}{(2!)(17!)} = 36$$

Therefore, four regressions are one-ninth of the total number of combinations.

8. Suppose that the system (65) is transformed into a new structure using a nonsingular matrix F.

$$G^*p_t^* + H^*d_t^* = u_t^*, \tag{a}$$

where $G^* = FG$, $H^* = FH$, and $u_t^* = Fu_t^*$. Writing out the reduced form corresponding to the transformed equations,

$$p_t^* = \pi^* d_t^* + v_t^*, \tag{b}$$

where $\pi^* = -G^{*-1}H^* = -(FG)^{-1}(FH) = -G^{-1}H = \pi$

$$v_t^* = G^{*-1}u_t^* = (FG)^{-1}(Fu) = G^{-1}u_t = v_t.$$

Thus both the original and the transformed structure have the same reduced form.

9. This is identical to Theorem 2.3.1 in Fisher (1966, p. 37).

10. If equation (54) holds identically, the system is exactly identified. Otherwise, it is overidentified.

11. There is a minor empirical problem with the estimation of the structural-form equations. That is, the risk-free rate is not

observable but may only be proxied by, for example, average T-bill rate. However, T-bill rates have been fluctuating in quite wide a range recently as shown below:

Period	T-bill rate
1976-81	8.9%
1962-81	6.1
1926-81	3.0

Source: Ibbotson and Sinquefield's (1982) Rates are geometric averages over the periods.

12. If the structural parameters are estimated from the corresponding reduced-form parameters when the system is exactly identified, the estimator is called indirect least squares (ILS) estimator. When the two-stage least squares method is applied to an exactly identified equation, the resulting estimates are the same as those obtained by the ILS method.

A direct estimation of the structural-form parameters seems to be formidable due to the complex linear relationships within and between equations. This will be left for the future research.

13. The covariance analysis and regression analysis with dummy variables are equivalent from the point of view of hypothesis testing of equality of the slopes and/or the intercepts. See Feng-Yao Lee (1974).

14. As specified in Hypothesis 2, only the slope change across equations is tested. The intercept is assumed to be zero.

15. If (65) is valid there will be no aggregation bias involved in simple linear aggregation and thus the whole system will become a pooling of time-series and cross-section data.

16. The SUR estimation and test of equality of slope coefficients were done through SYSREG procedure of SAS package.

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